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**On Stability in Competition:
Tying and Horizontal Product Differentiation**

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Abstract

We combine Hotelling's model of product differentiation with tie-in sales. Tie-in sales condition the sale of one good upon the purchase of another good. In equilibrium firms choose zero product differentiation. Due to the tying structure no firm can gain the whole market by a small price reduction. Then we address the following questions: Can a firm with monopoly power in one market leverage this power into another market where it faces competition. What is the effect from tying on the profits of the monopolist's rival. In our model this effect is ambiguous.

Keywords: Horizontal product differentiation, tie-in sales, leverage theory of tying, foreclosure

JEL-Classification Numbers: $D43, L11, L12, L13$

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1 Introduction

Among others, Hotelling recognized that sales volumes change gradually as firms change their prices. This observation stands in sharp contrast to the discontinuous consumer behavior for a homogeneous good used by Bertrand. To explain why firm specific demand is continuous in price differences, Hotelling (1929) introduced a model of horizontal product differentiation. Two firms offer a good that is identical in all respects except for one characteristic. By choosing this characteristic firms can differentiate their goods. Consumers derive different utility from consumption of the goods because their most preferred characteristic varies. The larger the distance between a consumer's preferred and a good's actual characteristic, the lower her utility for this good. Transportation costs measure this reduction in utility.

Hotelling assumes linear transportation costs. In equilibrium firms choose characteristics for their goods as close as possible to each other, a result known as the principle of minimum differentiation. However, this result is invalid. D'Aspremont, Gabszewicz and Thisse (1979) showed that no equilibrium in pure strategies exists if both sellers are close to each other. Close competitors can gain the entire market by a small price cut. So there is a discontinuity in the firms' profit functions as prices change.

We combine Hotelling's model with tie-in sales. Tie-in sales require consumers to buy a good as a condition for buying another good. Generally, tie-in sales come in two main forms: fixed or variable proportions. Bundling refers to goods that are tied together in fixed proportions. Sales with goods tied in variable proportions leave to the buyer to decide on the respective quantities[1]. In our model consumers purchase no more than one unit of the

[1] Definitions for tie-in sales and bundling vary in the literature. But a survey of definitions goes beyond the scope of this paper. Here we choose definitions that, hopefully, helps to understand the terms.

tied goods. For this reason, a tying firm sells its goods in fixed proportions. The distinction between variable and fixed proportions disappears. Therefore, we use tie-in sales, tying and bundling as synonyms.

The classic example for tie-in sales is IBM's tabulating cards case. IBM maintained a quasi-monopoly on tabulating machines. To use these machines the lessees needed punch cards. In the market for punch cards IBM faced competition. A further example is *Cablecom* in Switzerland. *Cablecom* is the only provider for pay per view television, their so called digital cinema. To receive the pay per view cinema, consumers must also subscribe for a television service with fixed program sequence. The market for such a service is competitive. *Cablecom* faces competition from the conventional television program and another provider, *Teleclub*, with fixed program sequence.

Both examples have in common that the tying firm dominates one market and faces competition in another market. Two other examples illustrate this situation. An often-cited example for tie-in sales is *Times-Picayune* which published a morning and an evening newspaper. Its sole competitor in the daily newspaper field was an independent evening paper. *Times-Picayune* bundled an advertisement in the morning paper with the advert in its evening issue. Take Microsoft's bundling of Media Player into its operating system as a further example. Microsoft has power in the market for operating systems whereas it faces competition in the market for digital players. Such examples motivate the widely used setting of a monopolist in one market competing with another firm in a second market. In the second market firms offer homogenous or given differentiated products. We modify this basic framework by modelling the second market in Hotelling's way: A television provider must compile a program out of many categories like, e.g., sports and documentaries. Thus, our model endogenizes firms' differentiation choices. The combination of horizontal product differentiation with tie-in sales results in zero differentiation. In equilibrium, the firms' competitively supplied goods are homogeneous. Yet no firm attracts the entire market by a small price reduction. The tying firm does not serve consumers with low valuations for the monopoly good. The non-tying firm cannot win the entire market with a price reduction such that its price is non-negative. Not all of the tying firm's consumers give up the monopoly good for a price reduction.

Our model and its outcome are closely related to the work by Carbajo, de Meza and Seidmann (1990). As is common in the tying literature they assume the bundling firm being a monopolist in one market and facing competition from another firm in another market. In the duopoly market the firms' goods are homogeneous. Firms compete in prices. The main finding is that imperfect competition creates a strategic incentive for bundling. Bundling alters the behavior of the monopolist's rival and reduces competitiveness in the duopoly market. Specifically, if the monopolist bundles, it no longer sells to all consumers. It is profitable to serve only consumers with high valuations for the monopoly good. This in turn causes the monopolist's rival to act less aggressively. If the monopolist does not bundle, Bertrand competition drives prices down to marginal costs. Because bundling itself allows product differentiation, the bundle and the competitively offered good alone are not homogeneous. Both firms can raise prices above costs.

Like Carbajo, de Meza and Seidmann, we find that bundling softens competition. This competition softening mechanism is responsible for equilibrium stability in our combination of Hotelling's model with tie-in sales. Goods in the duopoly market are homogeneous. The tying firm's profit function still exhibits a discontinuity. But the discontinuity is at a price which is not profit-maximizing because it is no longer profitable to serve all consumers.

By contrast, the profit function of the tying firm's competitor exhibits no discontinuity. The competitor cannot induce all consumers to give up the monopoly good for a small price reduction.

The leverage theory of tying is a prominent concern in the literature about tie-in sales. According to this theory, tying is an instrument to use monopoly power in one market to gain an advantage or reduce competition in another market. We examine the leverage hypothesis within our model by studying the tying effects on competition. This brings us close to Whinston's (1990) reexamination of the leverage theory. His setting also consists of a monopolist in one market who competes in another duopoly market. Unlike most models, Whinston assumes increasing returns to scale in the production process. He shows that tying may alter the market structure in the duopoly market depending on the size of fixed costs. Tying serves as a mechanism to reduce the sales of the monopolist's competitor. Such foreclosure may lower the competitor's profits below a "level that would justify continued operation". The monopolist can transfer its power into the competitive market to exclude the rival.

We extend our model so that it is in line with Whinston's setting and find similar results. We also find the ambiguous effect from tie-in sales on profits of the tying firm's competitor. If fixed costs are not too large, foreclosure does not exclude the tying's firm competitor. Tie-in sales can even raise the competitor's profits. In Whinston's model the ambiguity depends on the specification for the monopoly good valuations. But we specify these valuations. So, there is another reason for the ambiguity. The reason are per unit distance transportation costs. While Whinston assumes differentiated products and uses general demand functions, we explicitly model product differentiation. This different modelling introduces per unit distance transportation costs that are responsible for the ambiguity. To be more precise, it is the level of transportation costs that makes the effect ambiguous. If transportation costs per unit distance are low, the goods in the competitive market are better substitutes. Thus, competition increases and prices are low. The monopolist can prevent such fierce competition by tying. Tying reduces price competition because it induces the monopolist's rival to price less aggressively. Reduced competition also benefits the tying firm's competitor. For low per unit distance transportation costs the competition reducing effect can even overweigh the foreclosing effect. In this case, the tying firm's competitor also benefits from tie-in sales and collects higher profits.

The paper is organized as follows: In section 2 we set up the model. Next, we derive the demand functions and the equilibrium in section 3. We extend the model in section 4 to analyze the decision to tie and its effect on competition. In section 5 we conclude.

2 The Model

Consider two firms 1 and 2 and two markets A and B. Firm 1 is a monopolist in market A. It offers a non-differentiable good A. By contrast, firm 1 competes with firm 2 in market B. Both firms supply good B that is identical in all respects except one characteristic. A line with length one describes all possible values of this characteristic. The firms locate on this unit line. Let q_i, $i = 1, 2$, denote firm i's location. We assume that firm 1 cannot locate to the right of firm 2, i.e., $q_1 \leq q_2$. Unit and fixed costs are zero for both firms and both goods. We want to show and understand equilibrium existence in horizontal product differentiation with linear transportation costs and tie-in sales. Therefore, we focus on pure tying. Firm 1 only offers a bundle containing one unit of each good A and B.

There is a continuum of consumers with unit mass. Each consumer demands at most one unit of good A. The consumers have valuations r_A for A. Valuations r_A are uniformly distributed on the interval $[0, 1]$. Each consumer has unit demand for good B. We denote by β a consumer's address on the unit line. This address reflects consumer β's most preferred location or good characteristic. Let t be transportation costs per unit distance. Then a consumer incurs linear transportation costs $t|q - \beta|$ if her address differs from sales location q. Independently of r_A, all consumers have the same gross valuation r_B for B.

Consumers have two options: either they buy from firm 1 a bundle containing both products, or they do not buy good A at all and purchase only good B from firm 2. Irrespective of consumers' addresses firm 1 charges the mill price p_1 for the bundle. Likewise, firm 2 sells good B to all consumers at the same mill price p_2. Firms pass on total transportation costs to the consumers. Thus, consumers pay a generalized price consisting of the mill price and transportation costs. If a consumer with address β buys the bundle from firm 1 she has utility

$$v(\beta, q_1, p_1) = r_A + r_B - t|q_1 - \beta| - p_1.$$

If she only buys good B from firm 2 her utility is

$$v(\beta, q_2, p_2) = r_B - t|q_2 - \beta| - p_2.$$

The set-up gives rise to the following two stage game. In the first stage, the firms simultaneously choose their locations. In the second stage, the firms simultaneously set prices. We look for a subgame perfect equilibrium in pure strategies.

3 The Equilibrium

3.1 Demand Specification

First of all, we need the demand functions to find the game's equilibrium. Although we are close to Hotelling's framework, demand specification in our model is different from Hotelling. In the standard Hotelling model consumers base their purchasing decision upon the generalized price. Equating the generalized prices for the firms' goods determines the indifferent consumer. This indifferent consumer has an address in-between the firms' locations. Only one consumer is indifferent between buying from 1 or 2. In our model consumers can be indifferent although they have an address in one of the firms' hinterlands. To see why, consider all consumers with $\beta \leq q_1$. These consumers buy firm 1's bundle if it yields a higher surplus than consuming only good B:

$$r_A - t(q_1 - \beta) - p_1 \geq -t(q_2 - \beta) - p_2.$$

Rearranging terms, we see that consumers willing to pay more than the difference between the price difference and the transportation costs difference travel to firm 1:

$$r_A \geq (p_1 - p_2) - t(q_2 - q_1). \tag{1}$$

Consumers buy the bundle if their valuations for A satisfy condition 1. But the valuations differ. Some consumers do not find A attractive enough to purchase the bundle. Hence, not all consumers to firm 1's left buy from firm 1. Analogously, some consumers with $\beta > q_2$ value A high enough that they buy the bundle.

The criterion given by condition 1 has a further implication. With respect to transportation costs, consumers with $\beta \leq q_1$ assess only the distance between q_1 and q_2. For a consumer living to the left of firm 1 transportation costs from covering the way to q_1 accrue anyway, independent of the address. Suppose the unit line represents broadcast programs. A program at the left endpoint corresponds to purely sports selection. The opposite right endpoint stands for documentary themes only. Let the firms be at locations different than endpoints. Consumers who like sports very much are on the left of firm 1. These consumers must take a movement in direction documentary, independently of the television provider. Then, valuation r_A is the only variable that affects the buying decision. If the valuation exceeds some critical value, the consumer buys the bundle. The analogous reasoning holds for all consumers with $\beta > q_2$.

To identify the demand functions we divide the unit line into three regions as shown in figure 1. Region X contains consumers with $\beta \leq q_1$. All

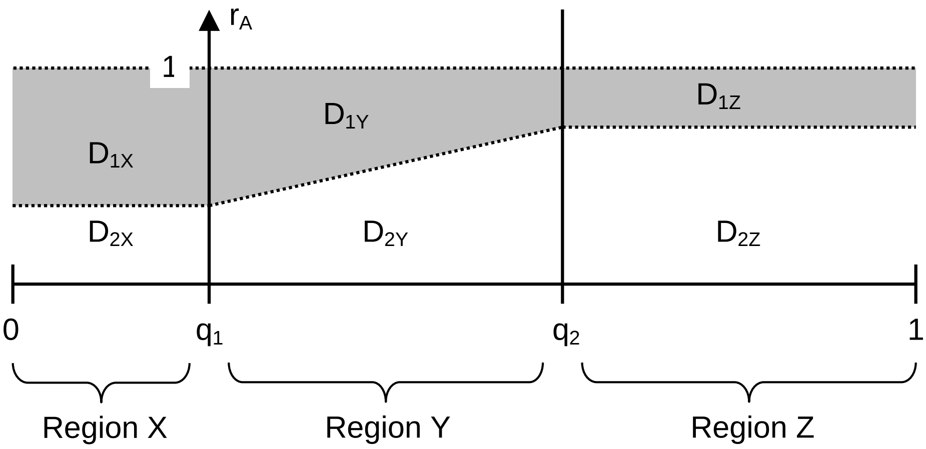

Figure 1: Demand Regions

consumers with $q_1 < \beta \leq q_2$ belong to region Y. In region Z lie all consumers to the right of firm 2's location, $q_2 < \beta$. Firm i serves demand D_{iR} in the respective regions $R = X, Y, Z$.

Demand D_{iX}: In region X consumers' addresses are irrelevant as argued above. Concerning transportation costs, only the difference between the locations weighted by the unit distance costs is important. Hence, all consumers with valuations satisfying condition 1 buy the bundle. The indifferent consumers are given by the equality

$$r_A = p_1 - p_2 - t(q_2 - q_1).$$

Demand in region X only consists of consumers to firm 1's left. The demand functions in region X are:

$$D_{1X} = q_1 Prob\left[r_A \geq p_1 - p_2 - t(q_2 - q_1)\right] = q_1 \left(1 - p_1 + p_2 + t(q_2 - q_1)\right),$$
$$D_{2X} = q_1 Prob\left[r_A < p_1 - p_{2B} - t(q_2 - q_1)\right] = q_1 \left(p_1 - p_2 - t(q_2 - q_1)\right).$$

Demand D_{iZ}: As in region X, only the difference between the sales locations affects the difference in transportation costs. Consumers located in region Z buy from firm 1 if they find A attractive enough. The valuation for good A must be at least as high as the sum of the price difference and the transportation costs difference,

$$r_A \geq p_1 - p_2 + t(q_2 - q_1).$$

In region Z firms serve the fractions

$$\begin{aligned} D_{1Z} &= (1 - q_2) Prob[r_A \geq p_1 - p_2 + t(q_2 - q_1)] \\ &= (1 - q_2)(1 - p_1 + p_2 - t(q_2 - q_1)), \\ D_{2Z} &= (1 - q_2) Prob[r_A < p_1 - p_2 + t(q_2 - q_1)] \\ &= (1 - q_2)(p_1 - p_2 + t(q_2 - q_1)). \end{aligned}$$

Demand D_{iY}: Consumers with addresses in region Y base their buying decision on valuation r_A and the generalized prices. All consumers with net utilities

$$r_A - t(\beta - q_1) - p_1 \geq -t(q_2 - \beta) - p_2$$

demand the bundle. Solving this decision rule for r_A yields the indifferent consumers' valuations depending on address, prices and locations:

$$\hat{r}_A(\beta, p_1, p_2, q_1, q_2) = p_1 - p_2 + t\,(2\beta - q_1 - q_2)\,.$$

The function $\hat{r}_A$ gives for each address the minimal valuation a consumer must have that she buys the bundle. Thus, unlike in region X and Z, address β affects the buying decision. A consumer who buys the bundle and has address far away from q_1 incurs high transportation costs. Whereas transportation costs are lower when buying from firm 2. The consumer only buys from 1 if consumption of A compensates for higher transportation costs. As $\partial\hat{r}_A(\beta)/\partial\beta > 0$ shows, addresses closer to q_2 require a higher r_A. Summing up $\hat{r}_A$ over region Y results in the fraction of consumers that buy from firm 2. Hence, the demand functions are:

$$\begin{aligned}
D_{1Y} &= q_2 - q_1 - \int_{q_1}^{q_2} \hat{r}_A(\beta)d\beta = (1 - p_1 + p_2)(q_2 - q_1), \\
D_{2Y} &= \int_{q_1}^{q_2} \hat{r}_A(\beta)d\beta = (p_1 - p_2)(q_2 - q_1)
\end{aligned}$$

Figure 2 illustrates the demand for firm 1's bundle and firm 2's good. The shaded area represents all consumers who have β-r_A-combinations such that they buy the bundle. Firm 2 serves demand corresponding to the non-shaded area.

Finally, we can state total demand for the bundle and for 2's good B. Total demand D_1 for the bundle consists in the sum of the respective fractions in each region R. Similarly, summing up each region demand for B gives total

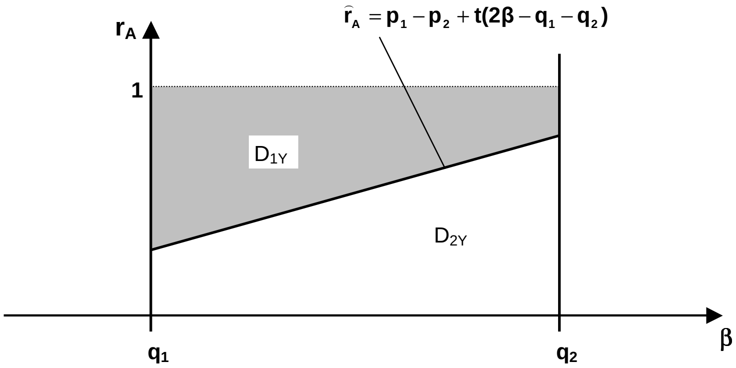

Figure 2: Demand in Region Y

demand D_2.

$$D_1 = D_{1X} + D_{1Y} + D_{1Z} = 1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2),$$
$$D_2 = D_{2X} + D_{2Y} + D_{2Z} = p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2),$$

3.2 The Firms' Behavior

In the second stage firms set prices given locations and the opponent's price. The firms' maximize profits

$$\pi_1 = p_1 D_1 = p_1 \left[1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2)\right],$$
$$\pi_2 = p_2 D_2 = p_2 \left[p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2)\right],$$

with respect to their prices. Maximizing and solving firms' profits with respect to prices gives the firms' reaction functions:

$$p_1(p_2) = \left(1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2)\right)/2,$$
$$p_2(p_1) = \left(p_1 + t(q_2 - q_1)(1 - q_1 - q_2)\right)/2.$$

Both price reaction functions are linear in the other firm's price and positively sloped. It follows that the reaction functions are well-behaved in the sense that they intersect only once. The intersection determines best response prices for the second stage at given locations. We can solve the system of equations given by the reaction functions for p_1 and p_2. This yields optimal prices for the second stage as functions of locations:

$$p_1^*(q_1, q_2) = (2 - t(q_2 - q_1)(1 - q_1 - q_2))/3$$
and
$$p_2^*(q_1, q_2) = (1 + t(q_2 - q_1)(1 - q_1 - q_2))/3.$$

Next, we turn to the first stage. Firms choose their profit-maximizing locations. Given their optimal pricing behavior, firms maximize profits

$$\pi_1 = [2 - t(q_2 - q_1)(1 - q_1 - q_2)]^2/9$$
and
$$\pi_2 = [1 + t(q_2 - q_1)(1 - q_1 - q_2)]^2/9$$

with respect to their locations. The firms' F.O.Cs. are

$$\partial\pi_1/\partial q_1 = 2t(2 - t(q_2 - q_1)(1 - q_1 - q_2))(1 - 2q_1)/9 = 0$$
and
$$\partial\pi_2/\partial q_2 = 2t(1 + t(q_2 - q_1)(1 - q_1 - q_2))(1 - 2q_2)/9 = 0.$$

Before solving for the optimal locations, note that outer derivatives resemble the price functions. Using the price functions we rewrite the F.O.Cs. in a more concise way :

$$\partial\pi_1/\partial q_1 = 2tp_1^*(q_1, q_2)(1 - 2q_1)/3 = 0$$
and
$$\partial\pi_2/\partial q_2 = 2tp_2^*(q_1, q_2)(1 - 2q_2)/3 = 0.$$

Then, we see that firms locate at 1/2. Otherwise, the firms choose locations such that profits are zero because prices are zero. If prices equal zero, firms are not profit-maximizing. The following proposition 1 summarizes the firms' equilibrium behavior.

Proposition 1 *In the Hotelling game with tie-in sales firms set equilibrium prices $p_1^* = 2/3$ and $p_2^* = 1/3$. Both firms locate at $q = 1/2$. Equilibrium profits are $\pi_1^* = 4/9$ and $\pi_2^* = 1/9$.*

In Hotelling's original model firms choose minimal differentiation. If product differentiation is minimal a small price reduction attracts all consumers. Therefore, the firms undercut each other. This effect does not occur in our model. No firm lowers its price although they choose the same location. To get the entire market firm 1 needs to lower its price below firm 2's price. Firm 1's profits when serving all consumers are $\pi_1 = p_2 - t(q_2 - q_1) - \epsilon$. If both firms locate at 1/2, firm 1 earns no more than $1/9 < \pi_1^*$. Hence, firm 1 does not change its price given its opponent's price.

If firm 2 wants to attract all consumers by a price reduction, it must compensate consumers for passing on good A. In the *Cablecom* example consumers switch to *Teleclub* only if the lower price compensates for giving up digital cinema. Unlike in Hotelling's standard model firm 2 does not win all consumers if it lowers its price by a small amount ϵ. Consumers with high valuations value digital cinema more than the price reduction. Because firms choose the same locations, their good B is homogenous. Then consumers prefer firm 2's good over the bundle if $r_A - p_1 \leq -p_2$. In equilibrium, the indifferent consumer has the valuation $\hat{r}_A = 1/3$. If firm 2 lowers its price by ϵ the equation $r_A = p_1^* - p_2^* + \epsilon$ identifies the new indifferent consumer. We see that firm 2's price reduction by ϵ increases demand for its good to the same extent. In this case firm 2 earns profits $(1/3+\epsilon)(p_2^*-\epsilon) < \pi_2^*$. Hence, firm 2 does not change its price given firm 1's price.

Proposition 1 shows that firms choose zero differentiation. This result corresponds to the homogeneity assumption in Carbajo, de Meza and Seidmann. They analyze a model with the same basic structure: Firm 1 is a monopolist in market A and faces competition by firm 2 in market B. The main finding is a strategic motivation for bundling. Firm 1 may bundle because it softens competition. By bundling firm 1 restricts itself. It only sells to consumers with high valuations for A. Thus, firm 2 sets a price above marginal costs. Both firms earn positive profits on the homogeneous good. Tying reduces competition that otherwise prevails in the duopoly market because it differentiates firms' products. This differentiation resembles vertical product differentiation. Thereby, the monopolistic good serves as surrogate for, e.g., quality. The same competition softening effect is responsible for equilibrium stability in our model. Because consumers have different valuations for A, or quality, firm 2 cannot attract its rival's consumers by a small price reduction. Firm 1 only serves consumers with high valuation for A. The discontinuity of firm 1's profit function is at a price which does not maximize profits. Because firm 2 cannot induce all consumers to buy only good B its profit function does not exhibit a discontinuity here.

4 Firm 1's Tying Decision and its Effect on Competition

To analyze firm 1's decision to tie its goods we extend our game to four stages. This extension also provides insights about the effects from tying on competition. We refer to this new game as the "decision game". Let us stick to the same setting as proposed in section 2. In addition we assume that supplying good B involves fixed costs K. Fixed costs are such that both firms are active in market B whenever firm 1 does not bundle[2]. The (new) sequence of events is as follows:

- Stage 1: Firm 1 once and for all chooses to offer its goods in a bundle or separately.
- Stage 2: Firm 2 once and for all decides whether to be active in market B or not.
- Stage 3: Active firms choose their locations.
- Stage 4: Firms simultaneously set prices.

We depict the timing for the decision game in figure 3.

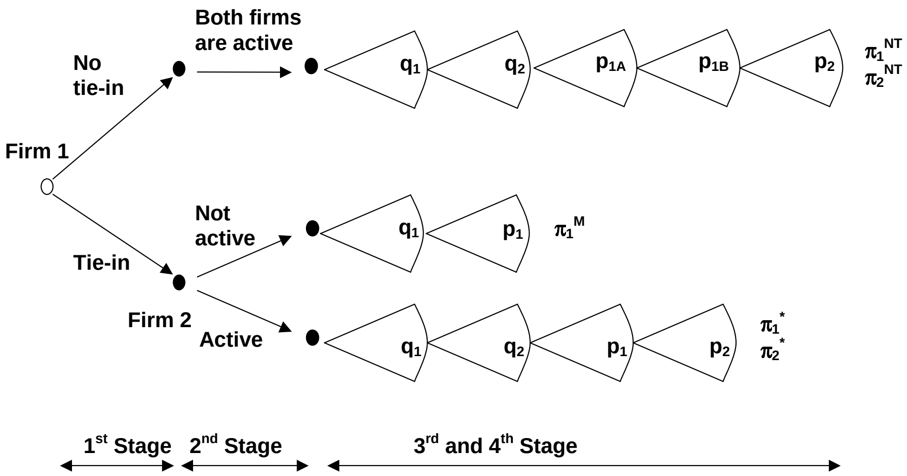

Figure 3: Timing of the Decision Game

If firm 1 does not bundle, market B is Hotelling's original model. In fact, no equilibrium in pure strategies exists. The most accurate way to handle

[2]We restrict attention to both firms being active in market B in the case without tying because we want to analyze the tying's effect on competition.

non-existence of equilibrium in pure strategies is calculating the (a) mixed strategy equilibrium. To the best of our knowledge, Osborne and Pitchik (1987) are the only ones who appeal to mixed strategies to solve Hotelling's original model. Osborne and Pitchik derive useful properties of the equilibrium strategies. Due to the complexity of the problem, however, they are not able to solve the game analytically and cannot specify equilibrium payoffs. But analytical specifications for firms' payoffs are necessary to solve our decision game. Thus, we use another way-out and limit our analysis to pure strategies. To obtain a pure strategy equilibrium we restrict the strategy spaces for firms' locations[3]. Firm 1 can only locate in the first and firm 2 in the fourth quartile, i.e., $q_1 \in [0, 1/4]$ and $q_2 \in [3/4, 1]$. The restriction ensures that firms cannot locate closer to each other than one half. As shown by d'Aspremont, Gabszewicz and Thisse, one half is the minimally required distance between the firms such that a pure strategy equilibrium exists. Adherence to this required distance does not affect the subgame with firm 1 bundling. For this subgame, we show later that the restriction of the strategy spaces does not affect firms' equilibrium prices and profits.

Note that firm 1 is always active in market B. If firm 1 bundles it always offers good B. If firm 1 does not bundle, the above assumption for K ensures that both firms supply B. So, we can study if tying can be an instrument to foreclose sales in, and thereby monopolize, a competitive market.

Again, we use subgame perfect Nash as equilibrium concept. To solve the decision game we need firms' behavior in the three subgames: Firm 1 does not bundle, firm 1 bundles but firm 2 is inactive and firm 1 bundles but firm 2 is active. All three subgames consist of the third and fourth stage of the decision game. We devote the next three subsections to identify equilibrium

[3]This approach is not new. See, e.g., Economides (1984, 1986), Hinloopen and Marrewijk (1999) or Posada and Strauma (2004)

outcomes in the location-then-price subgames.

The Subgame without Tie-in Sales

If firm 1 does not condition the purchase of good A on purchasing good B, the two markets are unrelated. Because the markets are not related and both firms are active, market B corresponds to Hotelling's model. The outcome in the price setting stage for Hotelling's two stage game does not change in our model. We can adopt his results for the fourth stage. In the fourth stage firms set prices $p_{1B} = t(2 + q_1 + q_2)/3$ and $p_2 = t(4 - q_1 - q_2)/3$ given the locations.

In the third stage the strategic variable for each firm is its location. Given the pricing behavior firms solve the maximization problems

$$\max_{q_1} \pi_{1B}(q_1) = t(2 + q_1 + q_2)^2/18 - K,$$
$$\max_{q_2} \pi_2(q_2) = t(4 - q_1 - q_2)^2/18 - K.$$

The derivative of profits shows that firms increase their profits by moving toward each other, $\partial\pi_{1B}/\partial q_1 > 0$ and $\partial\pi_2/\partial q_2 < 0$. Firms locate as close as possible to each other as strategy spaces allow, as expected due to the principle of minimum differentiation. With minimally possible differentiation firms set the same price t for good B. At same prices both firms serve half the market. In the subgame without tying the firms earn the same profits $\pi_{1B}^{NT} = \pi_2^{NT} = t/2 - K$ in market B.

Firm 1 is monopolist in market A. In this market consumers have different valuations for one unit of good A. Altogether, they form a downward sloping demand function for A given by $D_A(p_A) = Prob[r_A \geq p_A] = 1 - p_A$. Firm 1 maximizes its profits by charging $p_A^{NT} = 1/2$ and earns profits $\pi_A^{NT} = 1/4$ on good A.

Lemma 1 *In the subgame without tie-in sales firm 1 sets the price* $p_A^{NT} = 1/2$ *in market* A*. Both firms set the same prices* $p_{1B}^{NT} = p_2^{NT} = t$ *in market* B*. Firm 1 locates at* $q_1 = 1/4$ *and firm 2 at* $q_2 = 3/4$ *in market* B*. Overall profits are* $\pi_1^{NT} = (1+2t)/4 - K$ *and* $\pi_2^{NT} = t/2 - K$*.*

Remember that fixed costs are such that both firms sell B in case of no tying. Because firm 2 has the lowest profits, $\pi_2^{NT} \geq 0$ determines a lower bound for t. By this condition, per unit distance transportation costs have to be no smaller than twice the fixed costs, i.e., $t \geq 2K$.

The Subgame with Inactive Firm 2 and Tie-in Sales

Firm 1 is a monopolist for both goods A and B if firm 2 is inactive. Then, firm 1 earns monopoly profits π_1^M. We do not calculate these monopoly profits. All we need to know is that monopoly profits are higher than profits whenever firm 2 is active. For concreteness, consider the following intuitive argument. Firm 1 is monopolist and uses tie-in sales. Consumers can only buy the bundle. Certainly, firm 1 sets a monopoly price higher than prices when it faces competition by firm 2. But suppose instead that firm 1 has two choices for the price. Either it sells the bundle at a price equal to p_1^*, the bundle price if firm 2 is active. Or, it charges a price corresponding to the sum of prices for A and B without tying. If firm 1 chooses between these two possibilities it picks the price that yields higher profits. Independently of the price, demand is higher than under competition. Because demand is higher firm 1's profits increase, even if it sells at competition prices. By monopolizing market B firm 1 earns higher profits than under competition.

The Subgame with Active Firm 2 and Tie-in Sales

The third subgame is the game studied in section 3. However, we must reanalyze the firms' behavior because we restrict the sets of locations. Firms' behavior in the price setting stage remains unchanged. We can pick up the reexamination in the location stage. It is the third stage in the decision game. For the purpose of exposition, we restate the derivatives of firms' profits with respect to locations:

$$\partial\pi_1/\partial q_1 = 2tp_1^*(q_1, q_2)(1 - 2q_1)/3,$$
$$\partial\pi_2/\partial q_2 = 2tp_2^*(q_1, q_2)(1 - 2q_2)/3.$$

We see that $\partial\pi_1/\partial q_1 > 0$ and $\partial\pi_2/\partial q_2 < 0$ for non-negative prices. Again, firms locate as close as possible to each other. We have established:

Lemma 2 *In the Hotelling game with tie-in sales and restricted strategy space for locations firms set equilibrium prices $p_1^* = 2/3$ and $p_2^* = 1/3$. Firm 1 locates at $q_1^* = 1/4$ and firm 2 at $q_2^* = 3/4$. Equilibrium profits are $\pi_1^* = 4/9 - K$ and $\pi_2^* = 1/9 - K$.*

Now, let us analyze the second stage of the decision game. In this stage firm 2 decides whether to be active in market B. According to our assumption for K firm 2 is always active if firm 1 does not bundle. This is not true whenever firm 1 bundles. If firm 1 bundles firm 2 is active in market B as long as $\pi_2^* = 1/9 - K \geq 0$. Thus, firm 2 is active for $K \leq 1/9$. If $K > 1/9$ firm 2's overall profits are negative. Firm 2 does not serve enough consumers to cover its fixed costs at the profit-maximizing price. Scale economies are too small and consequently firm 2 remains inactive.

Given firm 2's behavior in the second stage we turn to the first stage. In the first stage firm 1 decides about using tie-in sales or not. The decision whether to tie depends on profits under tying and no tying. For $K \leq 1/9$, that is firm 2 is active, firm 1 engages in bundling if

$$\pi_1^* = 4/9 - K \geq (1 + 2t)/4 - K = \pi_1^{NT}.$$

So, firm 1 uses tie-in sales for t small enough, i.e., $t \leq 7/18$. If $t > 7/18$ firm 1 does not bundle. In case K exceeds 1/9 firm 2 is inactive if firm 1 offers the bundle. Then, firm 1 always ties its goods because it can monopolize the market B. As a tying monopolist in both markets firm 1's profits are always higher than without tying. We summarize the firms' behavior in the decision game by proposition 2 and represent it in figure 4.

Proposition 2 *In the decision game, firm 1 bundles*

· *if $K \leq 1/9$ (i.e., firm 2 is active) and transportation costs per unit distance are small enough (i.e., $t \leq 7/18$). The firms set prices and earn profits given by lemma 2.*

· *if $K > 1/9$ (i.e., firm 2 is inactive). Firm 1 earns monopoly profits π_1^M.*

Otherwise, firm 1 does not bundle and the firms' prices and profits are given by lemma 1.

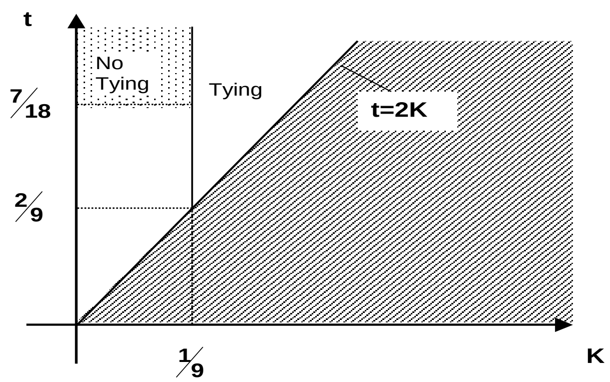

Figure 4: Firm 1's Tying Decision

Our analysis shows that tying forecloses firm 2's sales. In the no tying case firm 2 serves half the market whereas one third in case of tying. This sales foreclosure can make operation for firm 2 unprofitable. Here, firm 2 does not operate for $K > 1/9$. By excluding firm 2, firm 1 alters structure in the tied good market. Firm 1 forecloses enough sales to keep its rival inactive and thereby monopolizes the originally competitive market. For sufficiently large fixed costs tie-in sales are indeed an instrument to eliminate competition.

But as proposition 2 shows, firm 1 cannot always exclude its competitor from the market. If fixed costs are small, i.e., $K \leq 1/9$, firm 2 is active. However, tying can still be optimal for firm 1. This is true for $t \leq 7/18$. As lemma 1 shows, non-bundling prices are lower in the competitive market the lower t is. A decreasing t increases substitutability between goods because transportation costs are lower. Higher substitutability intensifies price competition. Firm 1 circumvents such intense competition by using tie-in sales. Its demand increases and prices become independent of t.

Tying always lowers firm 2's sales. This sales foreclosure leads to firm 2's exclusion if K is large enough. However, for small K, the foreclosure effect is not strong enough to keep firm 2 inactive. In this case the effect from tying on firm 2's profits is ambiguous and depends on t. Tying reduces intense price competition that occurs if t is small. If $t \leq 2/9$ tie-in sales entail higher profits for firm 2 compared to a non-tying regime:

$$\pi_2^* = 1/9 - K \geq t/2 - K = \pi_2^{NT} \quad \text{for} \quad t \leq 2/9.$$

Figure 4 depicts these per unit transportation costs by the upper triangle at the origin. For all other t firm 2's profits decrease whenever firm 1 ties. Thus, for t being small, firm 2 also profits from bundling although demand for its good is lower. The higher price under bundling can overcompensate the lower demand.

To get our ideas across we provide an example. Consider a situation with $t = 1/5$ and $K = 1/20$. Note that these parameter values are consistent with $t \geq 2K$. Because $t = 1/5 < 7/18$ firm 1 engages in tying. The bundle sells at a price $2/3$. If firm 1 sells the goods individually the sum of prices is

$1/5 + 1/4 = 9/20 < 2/3$. Tying allows firm 1 to charge a higher price than untied selling. But firm 2 also profits from tying via reduced competition. Without tying firm 2 sets a price $1/5$ and has profits $1/20$. Under tying firm 2's price is $1/3 > 1/5$ and profits amount to $11/180 > 1/20$.

The result that tying need not always result in excluding firm 2 or lowering its profits is consistent with Whinston's findings. He names two distinct reasons why commitment to tying may fail to lower firm 2's profits. First, good A may be unattractive. In Whinston's model tying creates an incentive to price more aggressively. The less attractive consumers find A the more aggressively prices a bundling firm. With aggressive bundle prices the margin between bundle price and unit costs is rather low. In this case firm 1's monopoly in market A is too weak for bundling to be an exclusionary threat. Because enough consumers find good A unattractive bundling rather helps than hurts the tying firm's rival. But unit costs and distribution for consumers' valuations for A do not vary in our model. Therefore, Whinston's first reason is not the cause for ambiguous effects from tying on firm 2's profits. The second reason is similar to the mechanism described in section 3 that leads to a stable equilibrium. Bundling transforms the homogeneous market into a setting comparable to vertical differentiation. Clearly, consumers value the bundle more than good B alone. With heterogeneous valuations for A consumers differ in their valuation for the bundle. According to Whinston, effects from bundling on firm 2's profits depend on how large this valuation difference is. We assume a specific distribution for r_A. Hence, the extent by which consumers' valuation differ is fixed. Therefore, magnitude of differences in consumers' valuation is not the reason for the ambiguity.

In our model, we find another reason for the ambiguous effect from tying on firm 2's profits. As shown above it is the level of per unit distance transportation costs t. Whinston assumes given differentiated products and uses general demand functions in the duopoly market. By contrast, we set it up in Hotelling's spirit. In this way, transportation rate enters the model. As long as firm 2 is active it can also profit from tying depending on t.

5 Conclusions

In his model of horizontal product differentiation Hotelling uses linear transportation costs. With linear transportation costs firms choose locations as close as possible to each other. But an equilibrium in pure strategies fails to exist. The reason is firms' proximity and the resulting undercutting. As d'Aspremont, Gabszewicz and Thisse show, firms must locate far enough from each other. Otherwise, firms have an incentive to attract consumers in the competitor's hinterland. If firms are not far enough, they gain the whole market by a small price reduction. This effect leads to non-existence of equilibrium in Hotelling's model.

In this paper we combine horizontal differentiation with tie-in sales. We adopt a widely used setting: Firm 1 is a monopolist in some market A and faces competition by firm 2 in another market B. In our model firms still choose as little product differentiation as possible, namely zero differentiation. But neither the bundling firm nor its competitor undercuts. The bundling firm does not undercut because it is no longer profitable to serve all consumers. Consumers have different valuations for the monopoly good. It turns out that the bundling firm only serves consumers with high valuations. Similarly, the bundling firm's competitor does not serve all consumers too. Because valuations for the monopoly good differ, the tying firm's competitor cannot gain the whole demand by a small price reduction. Attracting the whole market means for both firms decreasing profits. Thus, the incentive to undercut does not exist.

We extend our model to analyze the effects from tying on competition. In particular, we address the question if tying can alter the market structure. For this purpose we assume that entering the duopoly market involves fixed costs. In fact, the monopolist always forecloses its rival's sales by tying. But offering a bundle need not always be profitable for firm 1 compared to selling the goods independently. The profitability depends on per unit distance transportation costs and the possibility to exclude firm 2 from the market. Depending on fixed costs the foreclosure may be high enough that remaining in the market is unprofitable for the monopolist's rival. Complete elimination of competition is possible for large enough fixed costs. However, if fixed costs are not large enough, firm 2 remains active. Moreover, bundling may even increase firm 2's profits if per unit distance transportation costs are small enough.

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